

NAME: _____

Test Prep 4

Here is a problem where you can practice undetermined coefficients. If you finish this page, try the problems on the back. You have 10 minutes.

Find the solution to $y'' + 4y = 3t^2$ with $y(0) = 0$, $y'(0) = 0$

HOMOGENEOUS SOL'NS: $r^2 + 4 = 0 \Rightarrow r = \pm 2i \begin{cases} \lambda = 0 \\ \omega = 2 \end{cases}$

$$y_1(t) = \cos(2t), y_2(t) = \sin(2t)$$

PARTICULAR SOL'N: $Y(t) = At^2 + Bt + C$, $Y'(t) = 2At + B$, $Y''(t) = 2A$

$$y'' + 4y \stackrel{?}{=} 3t^2$$

$$2A + 4At^2 + 4Bt + 4C \stackrel{?}{=} 3t^2$$

$$(4A)t^2 + (4B)t + (2A + 4C) \stackrel{?}{=} 3t^2$$

$$4A = 3 \Rightarrow A = 3/4$$

$$4B = 0 \Rightarrow B = 0$$

$$2A + 4C = 0 \Rightarrow C = -1/2 A = -3/8$$

$$Y(t) = \frac{3}{4}t^2 - \frac{3}{8}$$

GENERAL SOL'N: $y(t) = c_1 \cos(2t) + c_2 \sin(2t) + \frac{3}{4}t^2 - \frac{3}{8}$

INITIAL CONDITIONS: $y(0) = 0 \Rightarrow c_1 - \frac{3}{8} = 0 \Rightarrow c_1 = \frac{3}{8}$

$$y'(t) = -2c_1 \sin(2t) + 2c_2 \cos(2t) + \frac{3}{2}t$$

$y'(0) = 0 \Rightarrow 2c_2 = 0 \Rightarrow c_2 = 0$

$$y(t) = \frac{3}{8} \cos(2t) + \frac{3}{4}t^2 - \frac{3}{8}$$

Extra problems for you to think about and attempt (not required for the official test prep):

1. Suppose you are solving a linear system $y'' + p(t)y' + q(t)y = 0$ with $t > 0$ and you find/guess three different solutions $y_1(t) = 2t^2 - 1$, $y_2(t) = 4 - 8t^2$, $y_3(t) = t^2$.

(a) Do $y_1(t)$ and $y_2(t)$ form a fundamental set of solutions?

$$W = \begin{vmatrix} 2t^2 - 1 & 4 - 8t^2 \\ 4t & -16t \end{vmatrix} = -32t^3 + 16t - 16t + 32t^3 = 0$$

NO!

(b) Do $y_1(t)$ and $y_3(t)$ form a fundamental set of solutions?

$$W = \begin{vmatrix} 2t^2 - 1 & t^2 \\ 4t & 2t \end{vmatrix} = 4t^3 - 2t - 4t^3 = -2t$$

YES!

(c) Write down the general solution to the equation.

Simplify your answer as much as possible.

$$y(t) = a_1(2t^2 - 1) + a_2 t^2 = (2a_1 + a_2)t^2 + (-a_1)$$

$$y(t) = c_1 t^2 + c_2$$

$$c_1 = 2a_1 + a_2$$

$$c_2 = -a_1$$

2. Find the general solution to $y'' + 4y' - 5y = 3 + 6e^t$.

(Here are two homogeneous solutions: $y_1(t) = e^t$ and $y_2(t) = e^{-5t}$.)

$$y(t) = A + Bte^t$$

$$y'(t) = Bte^t + Bte^t$$

$$y''(t) = 2Be^t + Bte^t$$

$$y'' + 4y' - 5y = 3 + 6e^t$$

$$2Be^t + Bte^t + 4Bte^t + 4te^t - 5A - 5Bte^t = 3 + 6e^t$$

$$\Rightarrow -5A = 3 \Rightarrow A = -\frac{3}{5}$$

$$\Rightarrow 6B = 6 \Rightarrow B = 1$$

$$y(t) = c_1 e^t + c_2 e^{-5t} - \frac{3}{5} + te^t$$